

Light propagation in non-linear electrodynamics

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Abstract

Working on the approximation of low frequency, we present the light cone conditions for a class of theories constructed with the two gauge invariants of the Maxwell field without making use of average over polarization states. Different polarization states are thus identified describing birefringence phenomena. We make an application of the formalism to the case of Euler-Heisenberg effective Lagrangian and well know results are obtained.

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I. INTRODUCTION

It is now a well established result that the velocity of the electromagnetic waves has its value dependent on the vacuum polarization states. Indeed, such polarization effects appear when a strong field (electrodynamical critical field: $E_{cr} = B_{cr} = m^2 c^2 / e \hbar \approx 1.3 \times 10^{18} \text{V/m} \approx 4.4 \times 10^{13} \text{G}$) is produced in some region of space. The most important consequence of this fact consists in the birefringence effect: the velocity of wave propagation depending on the wave polarization. A experimental method to detect the vacuum birefringence induced by a magnetic field was proposed in 1979 by E. Iacoppini and E. Zavattini [1]. In the experimental context, it is worth to mention the work of D. Bakalov *et al* [2], where optical techniques is used to detect birefringence in the presence of a strong magnetic field (PVLAS experiment). The theoretical description of nonlinear effects on light propagation was studied long before by Z. Bialynicka-Birula and I. Bialynicki-Birula [3], where was calculated the probability of the photon splitting in an external electromagnetic field. The same problem was extensively studied by S.L. Adler [4]. Other beautiful new results on vacuum polarization phenomena in nontrivial vacua, including curved spacetimes, can be found in the works of J.I. Latorre, P. Pascual and R. Tarrach [5], I.T. Drumond and S.J. Hathrell [6], G.M. Shore [7] and others. See for instance the works of K. Scharnhorst [8] and G. Barton [9], where the problem of photon propagation between parallel mirrors is worked out.

Recently, W. Dittrich and H. Gies [10], within the approach of the geometrical optics, derived the light cone conditions for a class of homogeneous nontrivial QED vacua using the rule of average over polarization states. They generalized some results previously obtained by Latorre and others, in particular the so called “unified formula” [5]. Indeed, such unified formula was identified by Latorre *et al* for several modified QED vacua and proved by Dittrich and Gies for certain cases. The use of the above mentioned average procedure excludes from their formalism the possibility to analyse the important phenomena of birefringence.

In this paper we deal with a class of Lagrangians depending on the two Lorentz and gauge invariants of the Maxwell field

$$L = L(F^{\mu\nu} F_{\mu\nu}, F^{\mu\nu} \tilde{F}_{\mu\nu}) \quad (1)$$

and working on the approximation of soft photons (the wavelength of propagating wave is large compared to the Compton wavelength), we present the light cone conditions for local theories of gauge invariant spin-one fields without

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making use of average over polarization states. Recent results by Dittrich and Gies are thus generalized in our approach, whose main contribution relies on the study of birefringence effects in a unified formalism. The polarization problem is worked out and the dispersion law is obtained, showing that there are a different polarization mode associated to each velocity of wave propagation. Finally, we apply the formalism to Euler-Heisenberg Lagrangian and some known results are derived in the context of the present formalism. We set the units $c = 1 = \hbar$.

II. NON-LINEAR SPIN-ONE THEORIES

Instead of calculating light cone conditions for each particular theory, we make use here of a general formalism, applicable to any Lagrangian based local theory describing gauge invariant spin-one fields that can be constructed with the two invariants of the Maxwell field. We denote the electromagnetic field strength by the anti-symmetric 2-rank tensor $F_{\mu\nu}$, and its dual is defined as

$$F_{\alpha\beta}^* = \frac{1}{2} \eta_{\alpha\beta}^{\sigma\tau} F_{\sigma\tau}. \quad (2)$$

Let us set the only two local and gauge invariant scalar fields F and G associated with $F_{\mu\nu}$ by

$$F = F^{\mu\nu} F_{\mu\nu} \quad (3a)$$

$$G = F^{\mu\nu} F_{\mu\nu}^*. \quad (3b)$$

In order to achieve more simplicity, we work in Minkowski spacetime employing a Cartesian coordinate system. Thus, the background metric will be represented by $\eta_{\mu\nu}$, which is defined by $\text{diag}(+1, -1, -1, -1)$. We defined the completely anti-symmetric tensor $\eta^{\alpha\beta\mu\nu}$ ($\eta^{0123} = 1$), and set the notation $L_X = \partial L / \partial X$, where the variable X stands for any monomial on the field invariants. The gauge invariant density of Lagrangian of electrodynamics is an arbitrary function of F and G :

$$L = L(F, G). \quad (4)$$

From the minimal action principle we get the equation of motion

$$\left(L_F F^{\mu\nu} + L_G F^{\mu\nu*} \right)_{,\nu} = 0 \quad (5)$$

where a comma denotes partial derivatives with respect to the Cartesian coordinates. Using relations $F_{,\nu} = 2F^{\alpha\beta} F_{\alpha\beta,\nu}$ and $G_{,\nu} = 2F^{\alpha\beta} F_{\alpha\beta,\nu}^*$ in equation (5) we obtain:

$$2N^{\mu\nu\alpha\beta} F_{\alpha\beta,\nu} + L_F F^{\mu\nu}_{,\nu} = 0 \quad (6)$$

where we introduced the 4-rank tensor $N^{\mu\nu\alpha\beta}$ through

$$N^{\mu\nu\alpha\beta} \doteq L_{FF} F^{\mu\nu} F^{\alpha\beta} + L_{GG} F^{\mu\nu*} F^{\alpha\beta*} + L_{FG} \left(F^{\mu\nu} F^{\alpha\beta*} + F^{\mu\nu*} F^{\alpha\beta} \right). \quad (7)$$

Additionally, the field strength $F_{\mu\nu}$ must satisfy the Bianchi identity.

Let us now turn our attention to the expressions which represent the light cone conditions for such general non-linear spin-one theory.

III. WAVE PROPAGATION

In this section we analyse the propagation of linear shock-waves associated with the discontinuities of the field in the limit of geometrical optics [11]. Let us consider a surface of discontinuity Σ defined by

$$z(x^\mu) = 0. \quad (8)$$

Whenever Σ is a global surface, it divides the spacetime in two distinct regions U^- and U^+ ($z < 0$ and $z > 0$, respectively). Given an arbitrary function of the coordinates, $f(x^\mu)$, we define its discontinuity on Σ as

$$[f(x^\alpha)]_\Sigma \doteq \lim_{\{P^\pm\} \rightarrow P} [f(P^+) - f(P^-)] \quad (9)$$

where P^+ , P^- and P belong to U^+ , U^- and Σ respectively. Applying the conditions [11] of discontinuities for the tensor field $F_{\mu\nu}$ and its derivatives we set

$$[F_{\alpha\beta}]_\Sigma = 0 \quad (10a)$$

$$[F_{\alpha\beta,\lambda}]_\Sigma = f_{\alpha\beta}k_\lambda \quad (10b)$$

where $f_{\alpha\beta}$ represents the discontinuities of the field on the surface Σ and k_λ is the wave propagation 4-vector. The discontinuity of the Bianchi identity yields

$$f_{\alpha\beta}k_\lambda + f_{\beta\lambda}k_\alpha + f_{\lambda\alpha}k_\beta = 0. \quad (11)$$

In order to obtain scalar relations, we consider the product of equation (11) with $F^{\alpha\beta}k^\lambda$ and with $F^{*\alpha\beta}k^\lambda$, which leads to

$$\zeta k^\lambda k_\lambda = -2F^{\alpha\beta}f_{\beta\lambda}k^\lambda k_\alpha \quad (12a)$$

$$\zeta^* k^\lambda k_\lambda = -2F^{*\alpha\beta}f_{\beta\lambda}k^\lambda k_\alpha \quad (12b)$$

where we have introduced the scalar quantities ζ and ζ^* as

$$\zeta \doteq F^{\alpha\beta}f_{\alpha\beta} \quad (13a)$$

$$\zeta^* \doteq F^{*\alpha\beta}f_{\alpha\beta}. \quad (13b)$$

In the same way, taking the discontinuities of the field equation (6), we get

$$f_{\beta\lambda}k^\lambda = -\frac{2}{L_F}N_\beta^{\mu\nu\rho}f_{\nu\rho}k_\mu. \quad (14)$$

Introducing relation (14) in equations (12) and making use of the useful identities between the tensor field $F_{\mu\nu}$ and its dual:

$$F_{\mu\alpha}^*F^\alpha{}_\nu \equiv -\frac{1}{4}G\eta_{\mu\nu} \quad (15a)$$

$$F_{\mu\alpha}F^{\nu\alpha} - F_{\mu\alpha}^*F^{*\nu\alpha} \equiv \frac{1}{2}F\delta_\mu{}^\nu \quad (15b)$$

we obtain

$$\zeta k^2 = \frac{4}{L_F}F^{\mu\nu}F^\tau{}_\mu k_\nu k_\tau (L_{FF}\zeta + L_{FG}\zeta^*) - \frac{G}{L_F}k^2 (L_{FG}\zeta + L_{GG}\zeta^*) \quad (16a)$$

$$\zeta^* k^2 = \frac{4}{L_F}F^{\mu\nu}F^\tau{}_\mu k_\nu k_\tau (L_{FG}\zeta + L_{GG}\zeta^*) - \frac{G}{L_F}k^2 (L_{FF}\zeta + L_{FG}\zeta^*) + \frac{2F}{L_F}k^2 (L_{FG}\zeta + L_{GG}\zeta^*) \quad (16b)$$

where $k^2 \doteq \eta^{\mu\nu}k_\mu k_\nu$.

In order to find the equation that represents the propagation of the field discontinuities, we seek for a master relation which should be independent of the quantities $f_{\alpha\beta}$ (that is, independent of ζ and ζ^*). There is a simple way to achieve such goal. We firstly isolate the common term $F^{\mu\nu}F^\tau{}_\mu k_\nu k_\tau$ which appears in both equations (16a) and (16b). The difference of these equations can thus be written as

$$\frac{\zeta k^2}{L_{FF}\zeta + L_{FG}\zeta^*} - \frac{\zeta^* k^2}{L_{FG}\zeta + L_{GG}\zeta^*} = -\frac{2Fk^2}{L_F} + \frac{Gk^2}{L_F} \left(\frac{L_{FF}\zeta + L_{FG}\zeta^*}{L_{FG}\zeta + L_{GG}\zeta^*} - \frac{L_{FG}\zeta + L_{GG}\zeta^*}{L_{FF}\zeta + L_{FG}\zeta^*} \right). \quad (17)$$

Assuming¹ $k^2 \neq 0$, we obtain an algebraic linear relation

¹Note that $k^2 = 0$ represents the standard propagation which occurs for the linear theory $L = -F/4$. We are interested here in the study of the possible deviations from this simple case.

$$\Omega_1 \zeta^{*2} + \Omega_2 \zeta \zeta^* + \Omega_3 \zeta^2 = 0 \quad (18)$$

between ζ and ζ^* , where we defined

$$\Omega_1 \doteq -L_{FG} + 2\frac{F}{L_F} L_{FG} L_{GG} - \frac{G}{L_F} L_{FG}^2 + \frac{G}{L_F} L_{GG}^2 \quad (19a)$$

$$\Omega_2 \doteq L_{GG} - L_{FF} + 2\frac{F}{L_F} L_{FF} L_{GG} + 2\frac{F}{L_F} L_{FG}^2 - 2\frac{G}{L_F} L_{FF} L_{FG} + 2\frac{G}{L_F} L_{FG} L_{GG} \quad (19b)$$

$$\Omega_3 \doteq L_{FG} + 2\frac{F}{L_F} L_{FF} L_{FG} - \frac{G}{L_F} L_{FF}^2 + \frac{G}{L_F} L_{FG}^2. \quad (19c)$$

Solving the quadratic equation (18) for ζ^* we obtain

$$\zeta^* = \Omega_{\pm} \zeta \quad (20)$$

with

$$\Omega_{\pm} = \frac{-\Omega_2 \pm \sqrt{\Omega_2^2 - 4\Omega_1\Omega_3}}{2\Omega_1}. \quad (21)$$

Using this solution into equation (16a) and assuming $\zeta \neq 0$ it results the following light cone conditions for spin-one fields:

$$\left[1 + \frac{G}{L_F} (L_{FG} + \Omega_{\pm} L_{GG}) \right] k^2 - \frac{4}{L_F} (L_{FF} + \Omega_{\pm} L_{FG}) F^{\mu\nu} F^{\tau}_{\mu} k_{\nu} k_{\tau} = 0. \quad (22)$$

In a similar way for equation (16b) we obtain

$$\left[\Omega_{\pm} - \frac{2F}{L_F} (L_{FG} + \Omega_{\pm} L_{GG}) + \frac{G}{L_F} (L_{FF} + \Omega_{\pm} L_{FG}) \right] k^2 - \frac{4}{L_F} (L_{FG} + \Omega_{\pm} L_{GG}) F^{\mu\nu} F^{\tau}_{\mu} k_{\nu} k_{\tau} = 0. \quad (23)$$

After some algebra, it can be shown that equation (23) is identical to (22).

The light cone conditions following from such procedure lead to two possible paths of propagation, according with the double solutions Ω_{\pm} . These two conditions are related to distinct polarizations modes, as we will see in the next section, indicating the possibility of birefringence. This effect depends upon the particular theory we shall consider.

In a more appealing form, we can present expression (22) as

$$k^2 = 4 \frac{L_{FF} + \Omega_{\pm} L_{FG}}{L_F + G(L_{FG} + \Omega_{\pm} L_{GG})} F^{\lambda\mu} F^{\nu}_{\lambda} k_{\mu} k_{\nu}. \quad (24)$$

Since $k^2 = \omega_o^2 - |\vec{k}|^2$, the phase velocity $\omega_o/|\vec{k}| \equiv v$ of the propagating light is found to be

$$v_{\pm}^2 = 1 - 4 \frac{L_{FF} + \Omega_{\pm} L_{FG}}{L_F + G(L_{FG} + \Omega_{\pm} L_{GG})} F^{\lambda\mu} F^{\nu}_{\lambda} k_{\mu} k_{\nu}. \quad (25)$$

This equation indicates that the familiar case $k^2 = 0$, which occurs for linear electrodynamics $L = -F/4$, is also possible for more complicated situations, as for those theories satisfying the relation $L_{FF} + \Omega_{\pm} L_{FG} = 0$. Solutions of this equation brings up the form of particular Lagrangians for which $k^2 = 0$, despite it was previously assumed that $k^2 \neq 0$. A simple solution of this equation can be obtained by setting $L = -F/4 + f(G)$, where $f(G)$ is an arbitrary function of the invariant G . Another example where $k^2 = 0$ occurs consists in the nonlinear Lagrangian of the N. Born and L. Infeld [12]. We are not interested here on the analysis of such theories.

The light cone conditions (24) can also be expressed in terms of the energy-momentum tensor of the non-linear field

$$T_{\mu\nu} = -4L_F F_{\mu}^{\alpha} F_{\alpha\nu} - (L - G L_G) \eta_{\mu\nu} \quad (26)$$

as

$$k^2 = -Q_{\pm} T^{\mu\nu} k_{\mu} k_{\nu} \quad (27)$$

where we defined the quantities

$$Q_{\pm} \doteq \frac{(L_{FF} + \Omega_{\pm} L_{FG})}{L_F^2 + GL_F (L_{FG} + \Omega_{\pm} L_{GG}) + (L_{FF} + \Omega_{\pm} L_{FG}) (L - GL_G)}. \quad (28)$$

Thus, in terms of the energy momentum tensor, the phase velocity for each propagation mode, corresponding to the solutions of Ω_{\pm} , are given by

$$v_{\pm}^2 = 1 - Q_{\pm} T^{\mu\nu} n_{\mu} n_{\nu} \quad (29)$$

where we introduced the quantity $n_{\mu} \doteq k_{\mu}/|\vec{k}|$ specifying the direction of wave propagation.

In the literature one usually makes use of an average over polarization states, which can directly be obtained from our formalism in the form $v = (v_+ + v_-)/2$. This represents the velocity of propagation for the average mode.

The results we had obtained in this section are applied for spin-one theories which is set by the Lagrangian function defined by (4), in the approximation of soft photons. The light cone conditions, here presented in terms of the field strength by (24) or else in terms of the energy-momentum tensor by (27), are useful in a variety of situations, and particularly for the study of birefringence phenomena. Let us now analyse in more details the problem of polarization.

IV. POLARIZATION

The most general decomposition for a skew-symmetric tensor is $f_{\alpha\beta} = (A_{\alpha}B_{\beta} - A_{\beta}B_{\alpha}) + (C_{\alpha}D_{\beta} - C_{\beta}D_{\alpha})$, where the vectors $A_{\alpha}, B_{\alpha}, C_{\alpha}, D_{\alpha}$ are arbitrary. For the case where $f_{\alpha\beta}$ is the wave propagation tensor given by equation (10b), for which equation (11) applies, it follows that the above decomposition simplifies to

$$f_{\alpha\beta} = a(\epsilon_{\alpha}k_{\beta} - \epsilon_{\beta}k_{\alpha}) \quad (30)$$

where a is the strength of the wavelet and ϵ_{β} represents the polarization vector.

The field equations impose restrictions on the possible states determined by such vector. Introducing (30) in (14) and assuming $a \neq 0$ we have

$$k^2 \epsilon^{\mu} = -\frac{4}{L_F} (N^{\mu\alpha}{}_{\nu\beta} k_{\alpha} k^{\beta}) \epsilon^{\nu}. \quad (31)$$

The $k^2 = 0$ case includes the linear propagation regime, where the polarization modes are well known. In this case equation (31) states that $N^{\mu\alpha}{}_{\nu\beta} k_{\alpha} k^{\beta} \epsilon^{\nu} = 0$. The $k^2 \neq 0$ case can be treated defining a symmetric tensor $Z_{\mu\nu}$ by

$$Z^{\mu}{}_{\nu} \doteq \delta^{\mu}{}_{\nu} + \frac{4}{L_F k^2} N^{\mu\alpha}{}_{\nu\beta} k_{\alpha} k^{\beta}. \quad (32)$$

With it we can write (31) as an eigenvector equation

$$Z^{\mu}{}_{\nu} \epsilon^{\nu} = 0. \quad (33)$$

The solutions of equation (33) (eigenvectors of $Z^{\mu}{}_{\nu}$) represent the dynamically allowed polarization modes. The definition of $N^{\mu\nu\alpha\beta}$, from equation (7), leads us to conclude that the tensor structure of $Z^{\mu}{}_{\nu}$ can be completely determined by the electromagnetic tensor, its dual and the wave vector k^{μ} . Hence, the general solution for the eigenvector problem can be achieved by expanding ϵ_{μ} as a linear combination of the following linearly independent vectors:

$$F^{\mu\nu} k_{\nu} \equiv a^{\mu}, \quad {}^*F^{\mu\nu} k_{\nu} \equiv {}^*a^{\mu}, \quad F^{\mu\alpha} F_{\alpha\nu} k^{\nu} \equiv b^{\mu}, \quad k^{\mu}. \quad (34)$$

Thus, the polarization vector takes the form

$$\epsilon_{\mu} = \alpha a^{\mu} + \beta {}^*a^{\mu} + \gamma k^{\mu} + \delta b^{\mu}. \quad (35)$$

Introducing (35) in (33) and taking the products, results

$$\begin{aligned} & \left\{ \alpha \left[\frac{L_F}{4} k^2 + L_{FF} a^2 + L_{FG} a^{\mu} {}^*a_{\mu} \right] + \beta \left[L_{FF} a^{\mu} {}^*a_{\mu} + L_{FG} {}^*a^{\mu} {}^*a_{\mu} \right] \right\} a^{\nu} \\ & + \left\{ \alpha \left[L_{FG} a^2 + L_{GG} a^{\mu} {}^*a_{\mu} \right] + \beta \left[\frac{L_F}{4} k^2 + L_{FG} a^{\mu} {}^*a_{\mu} + L_{GG} {}^*a^{\mu} {}^*a_{\mu} \right] \right\} {}^*a^{\nu} \\ & + \gamma [0] k^{\nu} + \delta \left[\frac{L_F}{4} k^2 \right] b^{\nu} = 0. \end{aligned} \quad (36)$$

In order to satisfy the above equation, the coefficients of each independent vector must be null, resulting in

$$\alpha \left[\frac{L_F}{4} k^2 + L_{FF} a^2 + L_{FG} a^\mu \dot{a}_\mu^* \right] + \beta \left[L_{FF} a^\mu \dot{a}_\mu^* + L_{FG} \dot{a}^\mu \dot{a}_\mu^* \right] = 0 \quad (37a)$$

$$\alpha \left[L_{FG} a^2 + L_{GG} a^\mu \dot{a}_\mu^* \right] + \beta \left[\frac{L_F}{4} k^2 + L_{FG} a^\mu \dot{a}_\mu^* + L_{GG} \dot{a}^\mu \dot{a}_\mu^* \right] = 0 \quad (37b)$$

$$\gamma = \text{arbitrary} \quad (37c)$$

$$\delta = 0. \quad (37d)$$

Therefore, from (30), γk_μ does not contribute to $f_{\alpha\beta}$, and we shall not consider it in any further. Using the relations (15) yields

$$a^\mu \dot{a}_\mu^* = \frac{1}{4} G k^2 \quad (38a)$$

$$\dot{a}^\mu \dot{a}_\mu^* = a^2 - \frac{1}{2} F k^2. \quad (38b)$$

Substituting these results in equations (37) and solving the system for k^2 we obtain the dispersion laws [3,14] for both polarization modes, which are described by the vectors ϵ_\pm^μ . Indeed, it can be shown that such dispersion relations are recovered from the light cone conditions (24) for all cases known in the literature, ensuring our previous statement concerning the relation between Ω_\pm with the two polarization states .

V. APPLICATION TO EULER-HEISENBERG LAGRANGIAN

In this section we apply the previous results to retrieve, from our formalism, a particular case of birefringence presented in the literature [2]. The most familiar non-linear case of electrodynamics is given by Euler-Heisenberg [13] effective action of QED²,

$$S = \int dx \left[-\frac{1}{4} F + \frac{\mu}{4} \left(F^2 + \frac{7}{4} G^2 \right) \right] \quad (39)$$

where dx stands for the volume measure of the spacetime, and the quantum parameter μ is defined by

$$\mu \doteq \frac{2\alpha^2}{45m_e^4}. \quad (40)$$

The light cone condition for each propagation mode can be directly obtained from expression (24) as m

$$k^2 = -8\mu F^{\alpha\mu} F_\mu{}^\beta k_\alpha k_\beta \quad (41a)$$

$$k^2 = -14\mu F^{\alpha\mu} F_\mu{}^\beta k_\alpha k_\beta. \quad (41b)$$

From equation (28), in the required order of approximation we obtain

$$Q_+ = 8\mu \quad (42a)$$

$$Q_- = 14\mu. \quad (42b)$$

Thus, in terms of the energy-momentum tensor,

$$k^2 = -8\mu T^{\alpha\beta} k_\alpha k_\beta \quad (43a)$$

$$k^2 = -14\mu T^{\alpha\beta} k_\alpha k_\beta. \quad (43b)$$

The phase velocities (25) corresponding to (41) reduce to

² The limit of weak-field from one loop approximated QED action.

$$v_+ = 1 - 4\mu F^{\alpha\mu} F_\mu^\beta n_\alpha n_\beta \quad (44a)$$

$$v_- = 1 - 7\mu F^{\alpha\mu} F_\mu^\beta n_\alpha n_\beta. \quad (44b)$$

These velocities correspond to two polarization states, showing that birefringence effect occurs in Euler-Heisenberg model, as it is well known in the literature. From the polarization sum rule we obtain

$$v^2 = 1 - 11\mu F^{\alpha\mu} F_\mu^\beta n_\alpha n_\beta. \quad (45)$$

This result was presented in the paper of G. M. Shore [7], and more recently by W. Dittrich and H. Gies [10].

VI. CONCLUSION

In this work we derived light cone conditions for a class of local and gauge invariant spin-one field theory constructed with the two invariants of the Maxwell field in the approximation of low frequency without making use of the average over polarization modes. In our formalism we took the different polarization states into account explicitly and one achieved a generalization of the wave propagation formula, which can be applied to the analysis of birefringence effects in a unified way. In order to illustrate its applications, we exhibited how to obtain some results of quantum electrodynamics concerning the wave velocity dependence on polarization states.

An interesting continuation of this work would be the analysis of the wave propagation equations as an effective modification of Minkowskian geometry for each polarization direction, due to non-linear effects. Investigations on this topic had already been performed (see reference [15], and more recently [16]). The use of the formalism presented here to the case of wave propagation in non-linear material media, also deserves further investigations.

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